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HYDRODYNAMIC INSTABILITIES

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Abstract:

Hydrodynamic instabilities occur when a control parameter (vertical temperature difference in Rayleigh–Bénard convection) exceeds a certain threshold. The generic Landau model of bifurcations may be applied to some instabilities. In the simplest case, the amplitude increases continuously and reversibly above threshold: in the Rayleigh–Bénard case, it is determined by a balance between the driving forces due to the density gradient and the diffusive effects opposing it. This chapter then discusses instabilities governed by centrifugal forces (Taylor–Couette) or surface-tension gradients (Bénard–Marangoni). This last case leads to the concept of sub-critical instabilities. The Kelvin–Helmholtz instability for parallel flows at different velocities is an example of open flows; the influence of the shape of the velocity profiles of these flows is described. Finally the stability of Couette and Poiseuille flows is discussed.

Keywords: Hydrodynamic Instabilities, Landau Model, Bifurcation, Rayleigh–Bénard Instability, Bénard–Marangoni Instability, Taylor–Couette Instability, Kelvin–Helmholtz Instability, Subcritical Instabilities.

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Introduction:

The foundations of hydrodynamic stability theory were laid down by Helmholtz, Kelvin, Lyapunov, Poincaré, Rayleigh, Reynolds, and Stokes in the nineteenth century. For more than a hundred years this subject attracted the attention of a great number of researchers. A vast body of literature on this subject exists and encompasses the work of mathematicians, physicists, engineers, astrophysicists, geophysicists, meteorologists, etc. Much background material on the classical approach to fluid stability can be found in the substantive general texts by Lin [1], Chandrasekhar [2], Joseph [3], Drazin and Reid [4], Swinney and Gollub [1], as well as several more specialized monographs, reviews, and collections of papers, such as Yudovich [2], Holm et al. [5], Arnold and Khesin [7], Godrèche and Manneville [4], Dikii [4], etc.

In addition to hydrodynamic stability, the twentieth century saw the birth of the sister discipline of magnetohydrodynamic stability which was developed in order to address important practical questions occurring, in particular, in thermonuclear fusion, astrophysics and dynamo theory, see, for example, Frieman and Rottenberg [70] and Lifschitz [117] The beautiful synergy and cross-fertilization between these topics was instrumental in accelerating advances in both of them. The key question of hydrodynamic stability theory can be formulated as follows: What happens to a given fluid flow under the influence of small disturbances. If the flow is robust under the influence of all small disturbances, it is called stable and can be expected to occur in nature. If there are perturbations which start to grow, we call the flow unstable and expect it to break up or otherwise change its character. These possibilities were experimentally demonstrated by Reynolds. The topic of stability is important both mathematically and for practical considerations. Generally, it is much easier to establish the instability of a given flow rather than to prove its stability. In fact, in this article we will argue that, in a certain sense, all nontrivial inviscid flows are unstable.

Discussion:

There are two well-established techniques for the analysis of hydrodynamic stability/instability, namely, spectral methods (normal modes) (see, for example, Chandrasekhar [3], Drazin and Reid [3]) and energy methods (see, for example, Arnold [4], Holm et al. [85], Arnold and Khesin [7], Vladimirov [1]). Recently, a modification of the spectral method called the pseudospectral method was proposed by Trefethen [165], Trefethen et al. [166], Waleffe [1] and others. These conventional techniques proved to be very useful for studying the stability/instability of some steady flows with relatively simple structure but their applicability to more complicated flows has proved to be limited. In this paper we emphasize a third technique, namely, the geometrical optics method which is capable of probing the instability of general classes of three-dimensional inviscid flows. (By its very nature this method cannot prove stability.) In contrast to spectral and energy methods, the geometrical optics method is specifically designed for studying highly localized short-wave perturbations of an arbitrary background flows. These perturbations are localized wave envelopes moving along the trajectories of fluid elements. The evolution of a particular envelope is governed by a characteristic system of ordinary equations along the relevant trajectory. In the language of geometrical optics the characteristic equations consist of the eikonal equation for the wave vector and the transport equation for the velocity amplitude. Needless to say, the system of ordinary differential equations is more tractable than the full system of partial differential equations governing the dynamics of general perturbations. Broadly speaking, the flow is unstable if the magnitude of the amplitude of the perturbed velocity grows in time without bound along at least one trajectory. As we will describe later, this observation produces an effective tool for detecting fluid instabilities.

For a system oriented upward, with gravity acting downward, the hydrodynamic stability of a thermal system will depend on both the thermal and the concentration drives acting on a REV, according to Eq. (7.24). Depending on the direction of the property gradients, both such drives may induce instability, leading eventually to turbulent flow. As such, unconditionally unstable situations are presented in Figure 7.2, where hotter fluid (Figure 7.2A) composed by a less dense mixture is positioned at the bottom of the fluid layer (Figure 7.2B). For positive βϕ and β_{C_ϕ} values, with negative gradients of $\langle \bar{T} \rangle^{i}$ and $\langle \bar{C} \rangle^{i}$, both drives expressed by Eqs (7.25) and (7.30) will give $G_{\beta}^{i} > 0$ and $G_{\beta_C}^{i} > 0$, respectively, causing positive source terms in ⟨k⟩ i in Eq. (7.31). If a heavier component is positioned at the top of this heated-from-below layer (Figure 7.2C), hydrodynamic instability will also occur and a source term will appear in Eq. (7.31). Then an initially laminar

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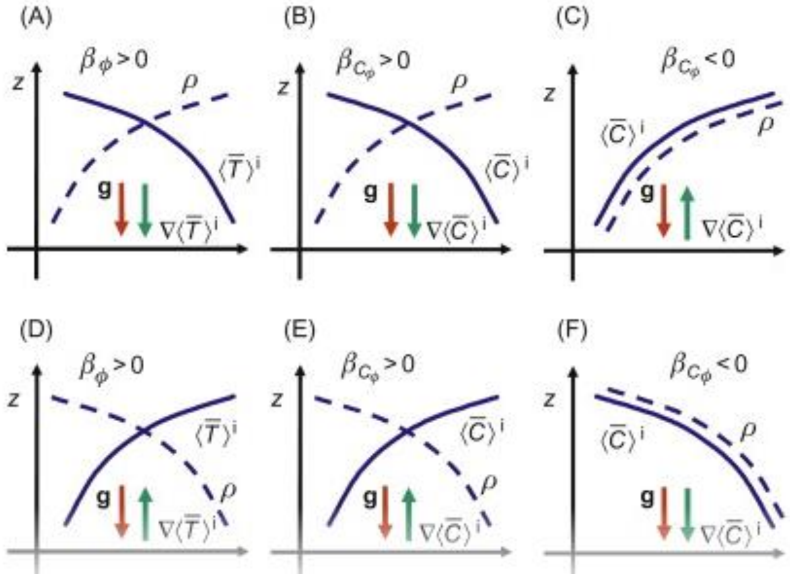
flow may undergo transition and become turbulent.



Methodology:

Hydrodynamic stability questions naturally arise in the astrophysical context and had attracted a lot of attention of mathematicians and astrophysicist alike. In order to build an adequate astrophysical stability theory, one needs to incorporate effects of rotation, compressibility, and gravity into the model. There is a vast and growing literature on the subject which can be traced back to the works of Riemann [1], Poincaré [4], Lyapunov, and others. Limitations of space make it impossible to give even a brief overview of all the relevant astrophysical stability problems and their solution methods. Accordingly, below we restrict ourselves to two representative examples. For a general discussion we refer the reader to the well-known books and review articles by Cox [2], Schutz [1], Tassoul [2], and Unno et al.

Analysis Procedure:

There are several basic approaches to determining the required submerged weight for a marine pipeline. One of them is use of AGA Program "LSTAB." It should be used in cases where the pipe is partially embedded or pre-trenched as the lift, drag, and inertia coefficients are adjusted for exposure. Regardless of the computer program selected, hydrodynamic stability analysis involves the following steps:

Conclusion:

There are two well-established techniques for the analysis of hydrodynamic stability/instability, namely, spectral methods (normal modes) (see, for example, Chandrasekhar [3], Drazin and Reid [3]) and energy methods (see, for example, Arnold [4], Holm et al. [85], Arnold and Khesin [7], Vladimirov [1]). Recently, a modification of the spectral method called the pseudospectral method was proposed by Trefethen [5], Trefethen et al. [6], Waleffe [6] and others. These conventional techniques proved to be very useful for studying the stability/instability of some steady flows with relatively simple structure but their applicability to more complicated flows has proved to be limited. In this paper we emphasize a third technique, namely, the geometrical optics method which is capable of probing the instability of general classes of three-dimensional inviscid flows. (By its very nature this method cannot prove stability.) In contrast to spectral and energy methods, the geometrical optics method is specifically designed for studying highly localized short-wave perturbations of an arbitrary background flow. These perturbations are localized wave envelopes moving along the trajectories of fluid elements. The evolution of a particular envelope is governed by a characteristic system of ordinary equations along the relevant trajectory. In the language of geometrical optics the characteristic equations consist of the eikonal equation for the wave vector and the transport equation for the velocity amplitude.

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