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Thermodynamic Study of Specific Black Hole Inspired by Generalized Entropy

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***Abstract**:*

The term "black hole" is of a very recent origin. It was coined in 1969 by the American scientist John Wheeler, as a graphic description to an idea that dates to at least two hundred years, to a time when there were two theories about light-the corpuscular theory and the wave theory. We now have a more concise theory in this respect-the wave-particle duality theory of quantum mechanics. John Michell, in 1783, was the first to provide a breakthrough, through his paper, Philosophical transactions of the royal society of London. In this paper, he pointed out that a star that was sufficiently massive and compact will have such a strong gravitational field that even light cannot escape-any light emitted from the surface of the star will be dragged back by the star's gravitational attraction before it could get very far. Michell suggested that there might be large number of stars just like this. Such entities which form black voids in space came to be known as black holes. Black hole thermodynamics is a rich subject, straddling both the classical and quantum aspects of gravity. The thermodynamic charges of a black hole such as entropy and temperature, while intrinsically quantum in nature, are related to classical attributes such as horizon area and surface gravity. Indeed, it was considering the classical response of a black hole to infalling matter that led Bardeen, Carter, and Hawking to make the link between black hole variations and laws of thermodynamics. More recently, our understanding of black hole thermodynamics and the interpretation of the various parameters has also been improving. The outline of this paper is as follows: First, we reviewed the various thermodynamic aspects of non-rotating and uncharged black holes like Reissner- Nordström, BTZ and Bardeen black holes. The findings were then incorporated to investigate, derive and compare thermodynamical parameters of Schwarzschild and Schwarzschild-AdS black holes. This study examines the thermodynamic features of the Van der Waals black hole within the context of three parameters of entropy. It is discovered that the parameters of black hole, including mass and temperature, deviate from those that are derived with the Boltzmann-Gibbs paradigm. Furthermore, the entropy parameter influences the behaviour of the Gibbs free energy by introducing thermodynamic instabilities, whereas the emission rate is mostly affected by entropy parameters at low frequencies. We provide expressions for thermodynamic quantities like pressure, temperature, heat capacity, Gibbs free energy, Helmholtz free energy, and isothermal compressibility. We investigate the phase structure of these solutions by analysing their heat capacity and Gibbs free energy. This investigation also examines the critical behavior and phase transitions of black hole. Additionally, we observe both local and global stability of black hole in the canonical ensemble for a variety of parameter values. We also investigate the effect of the entropy parameter and the sparsity of Hawking radiation on black hole evaporation.

Keywords: black Hole, The Expressions for Temperature (T), Gibbs Free Energy (G), Helmholtz Free Energy (F), Heat Capacity (C).

Introduction:

Black holes (BHs), among the greatest mysterious objects in the cosmos, are regions of intense spacetime curvature specified by an event horizon, a limit behind which neither matter nor light can escape. BH thermodynamics has advanced dramatically since the 1970s, with important innovations attributable to the revolutionary work of Bekenstein and Hawking. Hawking's area theorem [1] asserts that the total horizon area of BH cannot diminish over time during any physical activity that complies with the laws of classical physics. This theorem posits that BHs reflect thermodynamic characteristics analogous to entropy in thermodynamic systems. Jacob Bekenstein elaborated on this notion by suggesting that a BH's entropy is directly proportional to the area of its event horizon. The Bekenstein-Hawking entropy illustrates a significant correlation between the geometry of BHs and thermodynamic entropy [2]. Stephen Hawking's discovery of Hawking radiation further verified the comparison between BH thermodynamics and classical thermodynamics. Quantum physics at the event horizon influences this occurrence, leading to the emission of heat radiation from BHs. Thus, BHs possess a temperature, referred to as Hawking temperature, that is inversely proportional to their mass [3], [4], [5]. Entropy has become one of the greatest universal and key concepts in physics since the development of classical thermodynamics. Non-extensive thermodynamics suggests that entropy is subject to context and changes across theories, leading to a re-evaluation of its definition and purpose [6]. Hawking's radiation revealed that BHs radiate a black body spectrum with a temperature, establishing the Bekenstein-Hawking entropy as a key component of BH thermodynamics. The non-extensiveness of Bekenstein-Hawking entropy remains enigmatic, as classical thermodynamic entropy is additive and extensive. However, BH entropy defies this principle, leading to interest in alternative formulations Renyi and Tsallis entropies [3], [1], which are non-additive entropy models, help us understand non-extensive systems in new ways. Recent improvements, like Barrow entropy [2], Sharma Mittal [3], and Kaniadakis entropy concepts [4], [3], consider solving constraints in the classical formulations. Another factor to consider is the sparsity of Hawking radiation throughout the evaporation process [3],. The mean time between the emission of successive Hawking quanta significantly exceeds the natural timescale determined by the energy of the emitted quanta. This research deepens our understanding of deviations from the usual Boltzmann-Gibbs formalism by expanding on earlier work that investigated entropy corrections and their effects on BH thermodynamics. Corrections to the entropy-area relation of quantum fluctuations are discussed in [2]. This concept is relevant to our analysis of the effects of these corrections on the thermodynamic properties of the van der Waals (VDW) BH, namely its mass-temperature connection and phase diagram. As suggested in [3], the fact that entropy corrections could potentially have a significant effect on BH stability and phase transitions while studying compressibility, Gibbs and Helmholtz free energy, and heat capacity. Additionally, the sparse Hawking radiation is explained by [3], which leads us to wonder if there is an effect of entropy changes on the emission spectrum and the dynamics of VDWBH evaporation. Our results confirm stronger thermodynamics compared to standard BH models such as Schwarzschild and Kerr-AdS. The VDWBH, unlike the Schwarzschild BH, undergoes phase transitions like fluid systems, but with the transitions being induced by entropy instead of gravitational forces alone. Our analysis indicates that the dominant thermodynamic parameters, like entropy and pressure play an important role in establishing the phase structure of a VDWBH. However, angular momentum plays the central role in characterizing phase transitions and the stability of Kerr-AdS BHs. The results contribute greatly to BH thermodynamics by illustrating entropy-based generalized models enhance theoretical BH physics with correct modifications in terms of radiative properties, mass, temperature, and free energy.

Law for the Black Hole:

Law	In thermodynamics	For a black hole
Zeroth law	Temperature is uniform in a thermodynamic system in equilibrium	Surface gravity throughout the event horizon is uniform
First law	$dE = TdS +$ work terms	$dm = \frac{\kappa}{8\pi G} dA +$ work terms
Second law	$dA \geq 0$	$dA \geq 0$
Third law	$T = 0$ Cannot be achieved within a finite number of cycles	$\kappa = 0$ Cannot be achieved within a finite number of cycles

Along with Hawking's important finding of BH radiation, demonstrated a fundamental link within BHs and thermodynamic principles. The discovery that BHs are governed by thermodynamic principles marked a significant change in theoretical physics [4]. Numerous studies have explored the complex relationship between BHs and thermodynamics, expanding on the fundamental findings. BH thermodynamics focuses on the study of phase transitions. Davies was the first to identify phase transitions by measuring heat capacity at specific points [1]. The Hawking-Page transition marks a shift in the free energy of a BH, indicating a change in thermodynamic states [2]. Extensive research has been conducted on transitions between non-extremal and extremal BH states. Analogies with VDW systems have provided details regarding BH phase transitions and critical conduct, leading to a better understanding of their thermodynamic properties The comparisons have enhanced the analysis of BH thermodynamics, providing a more comprehensive view of crucial processes and phase transitions. The cosmological constant is transformed into a dynamic variable in the extended phase space approach to studying BHs in AdS space, similarly to the way pressure is treated in classical thermodynamics. This method allows for the definition of novel thermodynamic concepts, such as volume, which leads to a more comprehensive and exhaustive analysis of BH thermodynamics. Researchers can investigate critical phenomena, phase transitions, and stability conditions in BHs by employing the extended phase space framework. The connection among the thermodynamics of charged AdS BHs that are rotating and the VDW fluid is only qualitative, even though it is interesting. It's not always easy to see BH quantities like charge or angular momentum are linked together to fluid parameters a and b because the corresponding equations of state change. Unfortunately, this limitation continues to exist in all other challenging BHs, potentially in higher or lower dimensions, that exhibit qualitative VDW behavior. A comprehensive summary can be obtained by consulting the most recent inspection [72]. It is interesting to investigate the thermodynamic characteristics of VDWBH, especially with relation to entropy, which derives from their fascinating similarities to classical thermodynamic systems. Hawking radiation and BH entropies are among the many fascinating and exciting aspects of BH thermodynamics that have gained a lot of attention recently. It may offer a path toward a more profound understanding of quantum gravity. Incorporating the cosmological constant Λ into the fundamental law of BH mechanics gives us a consistent method to analyse the thermodynamics of the phase spaces of the (anti)-de Sitter spaces, to be more precise. BHs in de Sitter spaces are thermodynamically unstable and produce thermal radiation at varying temperatures. They can identify the relationship between the thermodynamic quantities on the two horizons, which could lead to an efficient method to study the thermodynamic quantities in de Sitter spacetime since studying the horizon thermodynamics of the de Sitter BHs independently makes the system thermodynamically unstable. A lot of research has been done on the critical behaviour of the effective thermodynamic quantities in de Sitter spacetime because of these arguments. A study of the effective thermodynamics of the rotating de Sitter spacetime has shown that it behaves in a way like that of VDWs [3], [4]. The effective thermodynamic quantities of the BH spacetime have recently been the subject of an investigation. We treat the cosmological constant in BH chemistry as a thermodynamic pressure, establishing a connection between BHs and thermodynamic systems. This framework extends the investigation of BH phase transitions, aligning with the VDW equation of state for real gases. BH thermodynamics are also changed by polymerization in the loop quantum gravity domain, which combines classical BH singularities with quantum ones. The intersection of these two concepts enables the study of critical phenomena and phase transitions of quantum-corrected BHs in relation to regular thermodynamic systems. The effects of polymerization significantly improve the microscopic structure of VDWBH, as these entities exhibit a phase transition comparable to that of liquid-gas behavior. The observational constraints provided by BH shadows, which have the potential to reveal signatures of quantum gravity, keep considerable importance. The incorporation of BH chemistry with polymerization yields new insights into the realms of space-time, thermodynamics, and quantum gravity [2]. The current study highlights significant differences in the thermodynamics of BHs when compared to the Boltzmann-Gibbs framework. The [3] study that analyzes the quantum Oppenheimer-Snyder model underlines quantum “corrections” to the mass energies of the BH which alters their thermodynamic properties, including mass, Hawking temperature, heat capacity, and stability. In the same way, the present findings on the VDWBH show that many of the thermodynamic attributes like pressure, temperature, Gibbs free energy, Helmholtz free energy, and compressibility deviate from the classical thermodynamic behavior. Both studies focus on the role of quantum phenomena and some other statistical mechanics in the evolution and stability of a BH. Other aspects, like the radiation of BHs, phase transitions, and stability from the thermodynamics point of view, are also studied extensively, supporting the theory of BHs and the application of traditional thermodynamic approaches. This paper concentrates on the thermodynamic properties of VDWBH, inspired by the connection between BH chemistry and the three-parameter entropy statistics in BH physics. This study is organized as follows: Section 2 describes the VDWBH and essential thermodynamic quantities in the three-parameter entropy domain. In Section 3, we address thermodynamic stability, including local stability, Gibbs free energy, Helmholtz free energy, the equation of state factor, the compressibility factor, emission energy rate, and the sparsity of Hawking radiation during the BH evaporation process. Section 4 summarizes and provides conclusions from the study discussed herein.

Discussion:

Section Snippets:

The three-parameter entropy in the perspective of Van der Waals black hole geometry

Here, we assume the general form of spherical symmetric spacetime $ds^2 = -\psi(r)dt^2 + \frac{dr^2}{\psi(r)} + r^2(d\theta^2 + sin^2\theta d\phi^2)$, the radial function takes the following form [88], [89]

$\psi(r) = -\frac{3\pi ab^2}{r(3b+2r)} - \frac{(4\pi ab)\log\left(\frac{r}{b}+\frac{3}{2}\right)}{r} + 2\pi a + \frac{r^2\left(\frac{3b}{2r}+1\right)}{l^2} - \frac{2M}{r}$. The metric is significantly simplified to the standard Schwarzschild AdS line element when $a = \frac{1}{2\pi}$ and $b = 0$. The VdWBH can be analysed as a thermodynamic generalisation of the Schwarzschild BH. The magnitude of intermolecular forces in the fluid is determined by the constant $a > 0$, …

Conclusion:

The VDWBH's thermodynamics are investigated in this study through the use of a three-parameter entropy framework. The expressions for temperature (T), Gibbs free energy (G), Helmholtz free energy (F), heat capacity (C), and emission energy rate in terms of horizon radius have been obtained by assigning the negative cosmological constant as a thermodynamic pressure.

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