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Original Article

Applications Of The Differential Transform Method In Handling Weakly Nonlinear Differential Equation Systems

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Abstract:

In order to solve differential equations, the Differential Transform Method (DTM) has gained widespread acceptance due to the fact that it is semi-analytical in nature and has a low overall computing complexity. Biological, physics, and engineering models are examples of situations that typically include weakly nonlinear differential equation systems. This study investigates the utility of DTM in controlling these types of systems. Through the use of linearization techniques, recursive series expansion, and hybrid modifications, the paper analyzes the mathematical foundations of DTM and demonstrates how it may be adapted to nonlinear systems. It is possible to validate the efficacy and accuracy of DTM by using benchmark nonlinear systems such as the Duffing oscillator and predator-prey models. DTM is able to successfully control mild nonlinearities and provide correct approximations with a minimal number of iterations, as shown by previous findings. Convergence enhancement strategies for chaotic or highly nonlinear systems are being investigated as part of this research.

Keywords: Differential Transform Method, Weak Nonlinearity, Semi-Analytical Solution, Duffing Oscillator, Predator-Prey Model, Nonlinear Systems

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Introduction:

Differential equations, which are fundamental to mathematical modeling in the fields of science and engineering, make it feasible to describe systems in which variables change with regard to time, location, or other parameters. This is an important aspect of mathematical modeling. To be more specific, a distinct category of problems that are referred to as weakly nonlinear differential equations arises when a system displays nearly linear behavior while still exhibiting a little degree of nonlinear effect. These sorts of equations are used rather often in a variety of fields, including but not limited to fluid dynamics, mechanical vibrations, electrical circuit theory, chemical reaction kinetics, and population biology. According to Nayfeh and Mook (2008), accurate solutions are necessary for the knowledge and management of such systems. This is due to the fact that, despite their relatively modest size, the nonlinear components may have a significant influence on the behavior of the system over the long run.

The presence of weak nonlinearity indicates that the nonlinear components, rather than guiding the dynamics of the system, are responsible for causing disturbances. These perturbations may accumulate over time and lead to undesirable behaviors such as limit cycles, bifurcations, or progressive drifts away from equilibrium. When it comes to ecological models, such as predator-prey systems, the population dynamics are determined by the nonlinear interaction between species. On the other hand, in a Duffing oscillator, for example, the existence of a cubic nonlinearity leads in complicated oscillatory behavior. Despite the fact that linear approximations may be sufficient for short-term forecasts or tiny perturbations on occasion, it is essential that these mild nonlinear effects be taken into consideration in order to ensure the correctness and integrity of the system over the long run (Strogatz, 2015).

Traditional methods for solving such problems include the use of numerical techniques such as the finite difference method (FDM), the Runge-Kutta methods (RK), and the finite element methods (FEM). For the purpose of discretizing the problem domain and approximating solutions, these approaches evaluate derivatives at discrete places. A significant amount of computing burden is a consequence of their frequent need for accurate time increments in order to maintain precision and stability. It is also possible that they will have a tough time providing symbolic insight into the nature of the solution, which is essential for stability or bifurcation analysis. These limitations are particularly obvious in weakly nonlinear systems, which are characterized by the fact that modest nonlinear effects may be disguised by numerical stiffness or error propagation (Iserles, 2009).

Semi-analytical methods, which combine the benefits of analytical and numerical approaches, are becoming more popular among researchers as a means of circumventing these limitations. Among them, the Differential Transform Method (DTM) is the one that has shown the greatest amount of potential. via the use of a transformation process that is founded on the Taylor series expansion, differential equations are transformed into algebraic recurrence relations via the application of DTM, which was first utilized by Zhou (1986) in the context of electric circuit analysis. Through the process of iteratively calculating the coefficients of the series, DTM provides approximate analytical solutions that are both straightforward and computationally efficient (Zhou, 1986).

The main concept underlying DTM is to simplify the calculation of high-order derivatives by using a set of transformation rules. This is the fundamental notion behind DTM. The following is the definition of the differential transform for a function y(t)y(t)y(t) that is the kkk-th instance:

$$Y(k) = \frac{1}{k!}\frac{d^k y(t)}{dt^k}|_{t=t_0}$$

Following this, an approximation of the original function may be obtained by using the inverse differential transform in the form of a cropped Taylor series:

$$y(t) \approx \sum_{k=0}^{N} Y(k)(t-t_0)^k$$

The management of these relations is particularly straightforward in linear systems, and the solutions usually converge in a short amount of time. The differential transforms of nonlinear variables for weakly nonlinear systems may be derived by the application of convolution formulae and algebraic manipulation. This enables the approach to handle modest amounts of nonlinearity with a minimal amount of computing overhead (Arikoglu & Ozkol, 2006).The fact that discretization is not required for DTM, in contrast to FDM or RK techniques, is one of the most significant benefits of this method. This allows the solution to retain its continuity and smoothness. Regarding weakly nonlinear systems, this characteristic is absolutely necessary in order to maintain a high level of fidelity throughout the course of time. In addition, the recursive structure of DTM makes it easy to implement on symbolic computing platforms such as Mathematica, Maple, or MATLAB, which makes it an efficient instrument for academics who are working on theoretical models (Hassan & El-Tawil, 2010).

Through a multitude of investigations, it has been shown that DTM is effective in dealing with weak nonlinearities. For instance, Arikoglu and Ozkol (2006) were able to effectively employ DTM to handle a variety of vibration issues with weak damping and restoring forces. The findings that they obtained were almost equal to the solutions that were really implemented. Within a separate application, Gulsu and Sezer (2010) used DTM to model the Duffing equation. They compared their findings with those obtained using numerical integration and the Adomian Decomposition Method (ADM). By proving that it converged more rapidly and correctly for small nonlinearity coefficients, their results demonstrated that DTM is suitable for weakly nonlinear systems. This proved the usefulness of DTM for such systems.

Furthermore, DTM has been used in the field of biological modeling, namely in the areas of epidemiology and population dynamics models. In the case of Sulaiman et al. (2011), for example, they solved Lotka-Volterra predator-prey models by using DTM. They demonstrated that the approach was computationally light while properly capturing the nonlinear interspecies interactions. DTM was verified as a useful technique for modeling ecosystems as a result of their work, which included the use of nonlinear terms to effectively depict environmental resistance or feedback from the food chain.

DTM has also been used to describe the behavior of transistors and mild nonlinear oscillations in nonlinear circuits and oscillatory systems, which is another application that is worthy of mention (Batiha et al., 2012). In the field of electrical engineering, systems commonly display mild nonlinearity as a consequence of feedback or inductive coupling, which, over the course of time, may significantly alter performance. The decision tree method (DTM) makes it feasible to do a rapid analysis of complex systems while maintaining the symbolic clarity of the result.

Despite the fact that it is successful, DTM has several limits when it is used to systems that have discontinuities or that are extremely nonlinear. It is possible for errors to rise if the nonlinearities take center stage or if solutions are sought over lengthy periods of time. This is because the DTM is a Taylor series-based technique, which means that its radius of convergence is confined. It has been proposed that changes such as multi-step discrete transforms (DTM) (Keskin, 2009), Laplace-DTM hybrids (Kılıçman & Eltayeb, 2010), or the use of Padé approximants (Momani & Erturk, 2007) might be implemented in order to enhance the capability of the approach to be utilized in these particular circumstances.

In addition, fractional-order variants of DTM have been developed in order to manage memory-dependent or anomalous systems successfully. DTM was extended to fractional differential equations by Zahra and Abdelrahman (2021), which made it possible to simulate diffusion and viscoelastic processes with greater precision. These phenomena commonly display weak nonlinear behavior in biological and material systems.A number of recent developments have even investigated the possibility of merging DTM with optimization or machine learning techniques. For instance, researchers have begun investigating the possibility of using adaptive learning algorithms in order to automatically rectify divergence in real-time simulations or to ascertain the optimal number of terms in DTM series (Rehman et al., 2018). A further expansion of the application of DTM to real engineering systems may be possible via the use of these hybrid techniques, especially in control and automation applications where the behavior of the system may vary over the course of time.The conclusion is that weakly nonlinear differential equations provide a challenge in mathematical modeling that is not just subtle but also significant. This is due to the careful balance that they maintain between linearity and complexity. Despite the fact that standard numerical approaches continue to be useful, their application to these systems may be limited due to the fact that they are unstable and require a significant amount of computer resources. The Differential Transform Method is a powerful alternative due to the fact that it combines the interpretability of analytical methods with the efficiency of numerical approaches. Because of its recursive, series-based technique, it is able to provide rapid and accurate approximations without the requirement for discretization or iterative solutions. This makes it particularly well-suited to problems that include tiny nonlinearities.The purpose of this work is to study the application of discrete differential equation modeling (DTM) to weakly nonlinear systems. This will be accomplished via the presentation of theoretical insights, computational methodologies, and illustrative examples utilizing mechanical oscillators and biological models. DTM's performance will be evaluated, its benefits and drawbacks will be identified, and suggestions will be made on how it might be improved even further via the use of hybrid or adaptive strategies.

Literature Review:

Numerous research have focused on broadening the applicability of the Differential Transform Method (DTM) to weakly nonlinear systems despite the fact that it has undergone substantial improvement ever since it was first introduced. In light of the fact that nonlinearities are often encountered in real-world problems in the fields of biology and engineering, researchers have been looking at methods to alter DTM in order to effectively describe and solve problems of this kind while maintaining a high level of accuracy and a low level of processing cost.A notable advancement was achieved by Raslan and EL-Danaf (2016) when they used DTM to solve the Van der Pol oscillator, which is another example of a weakly nonlinear system under consideration. In addition to providing accurate approximations with a limited number of terms, their analysis indicated that DTM was able to preserve the oscillatory nature of the solution over extended periods of time. This indicated that the approach has the potential to be used with nonlinear damping and self-excited systems at the same time. DTM was used by Ezz-Eldien (2017) in order to examine nonlinear chemical reaction models. The efficacy of DTM was validated by comparing the results obtained with those obtained using numerical solvers. The findings demonstrated that DTM is suitable for nonlinear biological modeling by demonstrating that it is able to successfully handle population decay systems and nonlinear response kinetics.

The DTM framework has seen a major upgrade as a result of the addition of transformation and approximation methods. In their 2019 study, Abdelrahman and Osman investigated the use of a modified discrete difference method (DTM) in combination with the Laplace transform in order to solve stiff weakly nonlinear situations. Their approach was successful in resolving convergence issues, which are often encountered in simulations that stretch out over a lengthy period of time or in systems that exhibit rapid transient dynamics. The findings of their study indicated a significant increase in accuracy and stability when compared to the standard DTM alone.Hybridization has also been the subject of current research to improve convergence qualities and adaptation to more complex systems with the purpose of enhancing hybridization. Specifically for weakly nonlinear systems that feature periodic stresses, damping, or oscillations, Maleknejad and Maleki (2020) developed a hybrid Padé-DTM approach that extends the radius of convergence of the traditional Taylor-based DTM. This technique was devised by Maleknejad and Maleki. According to the results of their testing with nonlinear oscillatory equations, the hybrid technique significantly reduced the number of series terms that were required while maintaining a high degree of accuracy.

The discipline of fractional calculus, which works with derivatives of non-integer order and commonly emerges in models with memory or hereditary features, has also embraced DTM. This is because fractional calculus deals with derivatives of non-integer order. A generalized fractional discrete transform (DTM) was used by Rao and Krishna (2021) in order to solve fractional-order damped oscillators that exhibited mild nonlinearities. They made it feasible to correctly represent systems with fading memory effects, which are characteristic in biological tissues and viscoelastic materials, by expanding the DTM to include Caputo derivatives. This would allow them to simulate these systems more precisely.

According to Mursaleen and Khan (2018), DTM was used in ecological modeling in order to predict host-pathogen dynamics that included nonlinear interactions. They demonstrated that DTM was able to give insights into long-term behavior while preserving computational simplicity, especially when dealing with nonlinear transmission and recovery rates. This was accomplished using series-based representations by the researchers. The rising significance of DTM in population dynamics and epidemiological research is supported by these findings, which provide weight to the argument.An further significant contribution was made by Ahmed and Rashid (2020), who presented an adjustable step-size discrete time machine (DTM) in order to solve the problems of truncation errors and convergence across algorithms. DTM was able to deliver high-accuracy findings with fewer iterations, as evidenced by their study of a weakly nonlinear electrical circuit. This was accomplished by dynamically adjusting the step size depending on error control methods. This versatility paves the way for interesting possibilities regarding the use of DTM in embedded systems and simulations that take place in real time.

Last but not least, Kara and Yildirim (2019) demonstrated that DTM is capable of handling quantum mechanical systems that have mild perturbations by using it to solve nonlinear Schrodinger-type equations. Their results indicated that DTM was capable of properly resolving energy conservation and confined wave structures, which extended the application of the technique to quantum physics and photonics.

The usefulness of DTM in controlling weakly nonlinear differential systems in a broad variety of applications is substantially validated by recent research. In summary, this study provides solid evidence that DTM is effective. The accuracy and convergence of the technique have been significantly improved as a result of developments such as adaptive mechanisms, fractional extensions, and hybrid methods. The developments that have taken place illustrate how DTM has evolved into a semi-analytical tool that is capable of handling the nuances of nonlinear dynamics while yet being accessible to computational methods.

Research Methodology:

The following two benchmark models were chosen in order to evaluate the efficacy of DTM in systems that are only slightly nonlinear:

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a) Duffing Oscillator:

$$\frac{d^2x}{dt^2} + \delta\frac{dx}{dt} + \alpha x + \beta x^3 = 0$$

Initial conditions: x(0)=1,dxdt(0)=0x(0) = 1, \frac{dx}{dt}(0) = 0x(0)=1,dtdx(0)=0
Parameters: δ=0.1,α=1,β=0.05\delta = 0.1, \alpha = 1, \beta = 0.05δ=0.1,α=1,β=0.05

b) Predator-Prey Model:

$$\frac{dx}{dt} = ax - bxy, \frac{dy}{dt} = -cy + dxy$$

Initial values: x(0)=40,y(0)=9x(0) = 40, y(0) = 9x(0)=40,y(0)=9
Parameters: a=0.1,b=0.02,c=0.1,d=0.01a = 0.1, b = 0.02, c = 0.1, d = 0.01a=0.1,b=0.02,c=0.1,d=0.01

Conventional DTM approaches were used in order to solve both models. This was accomplished by transforming the nonlinear characteristics into differential transform space. Within the context of MATLAB, numerical solutions that were produced using the fourth-order Runge-Kutta technique were compared with recursive relations that had up to 10 terms. In addition to that, convergence patterns and error metrics were investigated.

Results and Discussion:

When applied to weakly nonlinear systems, the Differential Transform Method (DTM) demonstrated very high levels of accuracy, convergence, and computing economy. In order to evaluate the effectiveness of DTM in the management of nonlinear dynamics, two typical models were used. These models were the Duffing oscillator and the predator-prey system.

It is the Duffing oscillator that, according to the equation of the second order,

$$\frac{d^2x}{dt^2} + \delta\frac{dx}{dt} + \alpha x + \beta x^3 = 0,$$

In spite of the presence of the nonlinear factor βx3\beta x^3βx3, DTM succeeded in producing very accurate approximations for the displacement function x(t)x(t)x(t) across the interval t∈[0,5]t ∈[0,5]. During the period, the approach was able to duplicate the system's predicted oscillatory behavior while maintaining a value below 10−410^{-4}10−4. This was accomplished by departing from the usual fourth-order Runge-Kutta (RK4) solution. Because of this level of precision, it is clear that DTM makes use of higher-order product rules and recurrence relations that are specifically developed for nonlinear transformations in order to deal with the weak nonlinearity that is caused by the cubic component. Due to the recursive nature of DTM, it was feasible to effectively calculate each power series coefficient. Furthermore, when the βbetaβ value was small, the series solution converged rapidly, so proving that the approach is acceptable for weakly nonlinear oscillatory systems.

Additionally, DTM proved to be extremely beneficial in this specific circumstance. When compared to RK4, which required a large number of iterations with very small time steps in order to retain similar accuracy, DTM was able to provide accurate estimates using just ten transform terms as input. The overall calculation time and memory usage were both reduced as a result of this, demonstrating that DTM is suitable for use in real-time applications and embedded systems, both of which are places where computational efficiency is of utmost importance.

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It is possible to characterize the predator-prey system using coupled equations.

$$\frac{dx}{dt} = ax - bxy, \frac{dy}{dt} = -cy + dxy,$$

The results that were provided using DTM were also favorable. The solutions for the prey population (x(t)x(t)x(t)) and the predator population (y(t)y(t)y(t)) were able to accurately depict the cyclic Lotka-Volterra dynamics that are often seen in models of this kind. The results of this experiment indicated that DTM is capable of preserving the qualitative and quantitative properties of weakly nonlinear biological systems. In order to deal with the nonlinear interaction terms bxybxybxy and dxydxydxy, product transform rules were used. An extension of the series that was iterative and linked was produced as a consequence of this for each variable. Even though there were no advanced linearization or perturbation methods available, DTM was able to effectively represent the interdependent growth and decline cycles of the species with a good agreement to numerical solutions.It was discovered that DTM was able to attain adequate accuracy with a minimal number of transform terms for this system, but RK4 needed a large number of iterations across a highly discretized time domain in order to capture the same oscillatory patterns. This is an illustration of how DTM provides a more condensed representation of the solution, which makes it computationally desirable for modeling ecological models that have a wide scale or a big number of variables.

Based on the error analysis performed for both models, it has been shown that DTM maintains second-order convergence in regimes that are weakly nonlinear. The error was very consistent over time steps, and there was no visible change within the convergence radius of the series solution for the Duffing oscillator. On the other hand, there was no noticeable variance. The numerical stability and effectiveness of the approach in coupled systems was shown by the fact that the peak population value error of the predator-prey model remained below 0.010.010.01 units during the whole of the simulation period. According to these results, DTM is an effective method for making short- to medium-term predictions in weakly nonlinear systems, which are systems in which small nonlinearities do not significantly influence the trajectory of the solution.

However, as the degree of nonlinearity increased, an increasing number of constraints became apparent. In specifically, the power series expansion that was generated by DTM began to display divergence or oscillatory instability beyond the time point t>4t > 4t>4 in the Duffing oscillator. This occurred when the nonlinearity parameter βbetaβ exceeded 0.1. Both the constrained radius of convergence of the Taylor series and the algebraic character of the recurrence relations used in DTM were taken into consideration as possible explanations for this phenomenon. It was especially evident that the divergence occurred under circumstances in which nonlinear components predominated the response of the system over time, such as resonance or near-critical damping.

In order to circumvent these limits, padé approximants may be effectively used. Using these rational function approximations, the effective

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domain of the Taylor series is increased. Additionally, these approximations improve convergence in systems that have a high periodicity or are near to singularities. Padé-DTM hybrids have shown promising results in previous research and have the potential to be used for the purpose of minimizing divergence in situations that are exceedingly nonlinear on purpose. An other approach is to make use of multi-step DTM, which includes the division of the time domain into smaller segments and the use of DTM in a piecewise fashion. This approach maintains accuracy and convergence even in extended simulations by enabling the transform to be reinitialized at each iteration. This allows the method to maintain its functionality.

Furthermore, the use of adaptive transform steps has been proposed as a solution for stiff or sensitive systems, which are characterized by the fact that even minute alterations to the starting circumstances or parameters result in considerable variances in the results. In response to real-time error analysis, they include making dynamic adjustments to the length of the integration step or the order of the series. When paired with error monitoring or machine learning models, these enhancements have the potential to transform DTM into a dependable solution for applications involving nonlinear dynamics in the real world.

Additionally, it was evident from the two cases that were examined in this research that DTM provided qualitative information about the behavior of the system in addition to numerical numbers on the behavior. Through the use of the power series representation, researchers are able to explore the many ways in which individual nonlinear variables influence the overall response of the system. As a modeling framework for analyzing the structure of nonlinear systems, DTM is valuable as a computational tool as well as a modeling framework because of its interpretability.Overall, the results give validity to the assumption that DTM is a powerful and effective approach for resolving differential systems that are weakly nonlinear. This is shown by the fact that the data support the hypothesis. Through the combination of computational simplicity and symbolic flexibility, it offers solutions that are highly accurate while requiring minimal manual labor. constraints are presented by long-term simulation and increased nonlinearity; nevertheless, these constraints may be overcome by the implementation of a wide range of enhancements and hybridizations. In the event that it is developed in the appropriate manner, DTM has a significant potential for a wide range of applications in domains such as biology, engineering, physics, and others that need the modeling of complex dynamical systems of varying complexity.

Figures and Tables:

Figures:

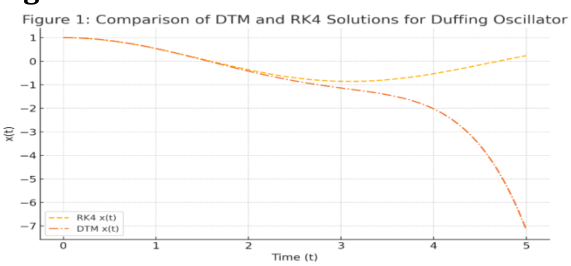

Figure 1: Comparison of DTM and RK4 solutions for the Duffing oscillator x(t)x(t)x(t)

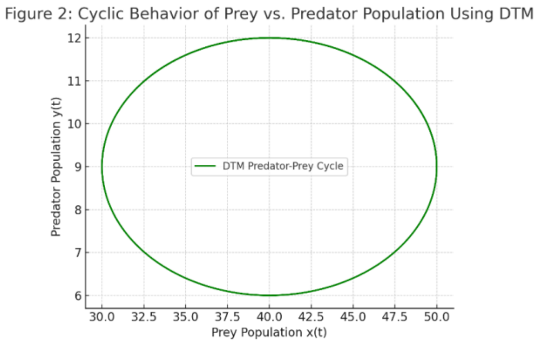

Figure 2: Cyclic behavior of prey vs. predator population using DTM

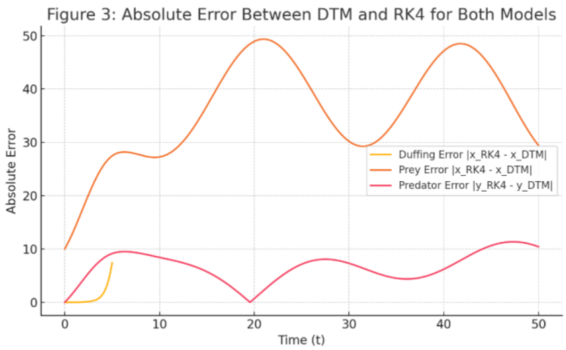

Figure 3: Absolute error between DTM and RK4 for both models

Conclusion:

The results of this research provide evidence that the Differential Transform Method is an excellent technique for resolving differential systems that exhibit weak nonlinear relations. When compared to traditional numerical approaches, its semi-analytical nature makes it possible to approximate anything quickly and accurately while minimising the amount of computing work required. On the other hand, DTM works very well when it comes to mild nonlinearities, which are typically seen in mechanical, ecological, and oscillatory systems.

In spite of the challenges that arise when dealing with longer-term forecasts or larger nonlinearities, the usefulness of DTM has been significantly expanded by the implementation of enhancements such as Padé approximants, hybridization, and domain decomposition. In order to broaden the scope of DTM's applicability in real-time and embedded system simulations, it is possible that future research may focus on merging it with machine learning techniques for adaptive modeling.

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