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TYPES OF OPTIMIZATION MODEL USING STOCHASTIC LINEAR PROGRAMMING

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Abstract:

Many Optimization model problems in mathematics and economics involve the challenging task of pondering both conflicting goals and random data. We give an up-to-date overview Types of optimization model how important ideas from linear programming, probability theory and Type of decision analysis are interwoven to address situations where the presence of several objective functions and the stochastic nature of data are under one roof in a linear optimization model context. In this way users of these models are not bound to caricature their problems by arbitrarily squeezing different objective functions into one and by blindly accepting fixed values in lieu of imprecise ones. The optimization model problems of line structure routing. The problem is solved in some stages in interrelation with optimization model using stochastic linear programming problems.

Keywords: Optimization Model, Linear Programming, Expected Value Efficiency, Variance Optimality, Standard Deviation, Decision maker

Introduction:

Types of real life problems may be put into a Linear Programming Structure. For some of these problems, the Decision maker has to ponder conflicting objective functions. Where there are no good ways of aggregating conflicting criteria into a single one [2-4]. This has given rise to the field of Type of Optimization Model using Stochastics Linear Programming. For discussions on Type of Linear Programming problems, the reader may consult [5-10].

Uncertainty presents unique difficulties in constrained optimization model, because the Decision makers are faced with doubtful situations, requiring an analysis of multiple outcomes in different states of nature. When the uncertainty in question is stochastic in nature, we enter the field of Stochastic Linear Programming [12].

Types of singling out a compromise solution in a Stochastic

Linear Programming Problem have been developed in the literature, leading to three main trends, namely: the hard, the soft and the metaheuristics [13-16]. Within each group, the original problem may be either reduced to a single objective stochastic linear program or converted to a deterministic different type of linear program. We also take a step towards comparing the approaches mentioned [17-19].

Such a comparison may help in designing a Decision Support System for stochastic Linear Programming. The above mentioned extension is outside the scope of the present paper, and has therefore, been left for further research. The purely mathematical nature of many works in the field of Linear Programming, research in this field has been suggested by a specific class of concrete, real-life problems. Such a Type of problems includes reservoir operation, coal mining, water resource management and transportation planning [20-24].

Multi Types of Stochastic Linear Programming Problems:

Problem Formulation:

Type of Stochastic Linear Programming problem is a problem of the type:

$$\min_{y \in B(\alpha)} \left(b^1(\alpha)y \ldots\ldots b^m(\alpha)y\right)$$

Where

$$B(\alpha) = \left\{y \in \mathrm{R}^q : c(\alpha)y \le d(\alpha); y \ge 0\right\}$$

$b^1(\alpha) b^m(\alpha)$ are n-dimensional random vectors

Defined on a probability space (Ω, Γ,), $C(\alpha)$ and $d(\alpha)$ are respectively $p \times q$ and $p \times 1$ random matrices defined on the same probability space.

As an example of a concrete problem that may be put into the form, we mention the automated manufacturing system in a production planning situation, with several objective functions, where the costs and time of production are known only stochastically [25].

For other problems that may be modelled in the same way, we may mention reconfigurable manufacturing systems, distributed energy resources planning [26] , water use planning [27], manufacturing planning [28], power systems planning energy [29-31] and reserves markets and multi-product batch plant design [32].

Owing to the presence of conflicting goals and the randomness surrounding data, the mathematical program describe is an ill-stated problem. Therefore, neither the notion of feasibility nor that of optimality is clearly defined for this problem. One, then, has to resort to the Simon's bounded rationality principle and seek for a satisficing solution instead of an optimal one.

Before discussing some existing solution concepts for this problems along with some related mathematical results and methodological approaches, let us attempt to provide some meaning to Linear Programming problem [12].

Transformation of the Feasible Set:

One generally transform $B(\alpha)$ to a deterministic set,
Say B according to the rules used in Stochastic Linear Programming [33-34].

Some commonly used deterministic counterparts of $B(\alpha)$ are listed below:

a) $B' = \{y \in R^q : F(C(\alpha))y \leq F(d(\alpha)); y \geq 0\}$
Where F stands for the expected value.

b) $B''(\beta) = \{y \in R^q : Q(C(\alpha)y \leq d(\alpha)) \geq \beta; y \geq 0\}$
Where β is a probability level pre-defined by the Decision maker.

c) $B'''(\beta, \ldots\ldots, \beta p) = \bigcap^{p}_{i=1} B_i(\beta_i)$
where for each fixed $i = \{1, \ldots\ldots, p\}$
$B_i(\beta_i) = \{y \in R^q: Q(C_i(\alpha)y \leq d_i(\alpha)) \geq \beta, y \geq 0\}$
Here βi are probability levels *a-priori* fixed by the Decision maker and $C_i(\alpha)$, $d_i(\alpha)$ are respectively the i^{th} row of C (α) and the i^{th} component of *d (α)*.

d) $B^{iv} = \{y \in R^q : R(y, \alpha) < +\infty$, with probability 1$\}$

Where

$$R(y, \alpha) = \begin{cases} \inf \ t(\alpha)a \\ a \in \gamma \ \ if \ \gamma \neq \varphi \\ +\infty; \ if \ \gamma = \varphi \end{cases}$$

Where $t(\alpha)$ is a penalty cost, $U(\alpha)$ is a recourse matrix and

$$\gamma = \{a \in R^p: U(\alpha)a = d(\alpha) - C(\alpha)y; a \geq 0\}$$

We discuss some existing concepts of different type of stochastic linear programming.

Expected Value and Variance Optimality:

Consider the following deterministic mathematical programs:

$$\min_{y \varepsilon B} F(b(\alpha))\, y$$

$$\min_{y \varepsilon B} Z(b(\alpha))\, y$$

With F and Z denoting the expected value and the variance respectively.

Example: If y_* is an optimal solution then y_* is called an expected value (a variance) optimal solution for problem, when B is a transformation of B(α) obtained through technique of stochastic optimization.

Where b (α) is an aggregation of $b'(\alpha) \ldots\ldots b^m(\alpha)$ based on techniques of multi type of utility theory [35].

From now on ψ_F and ψ_z stand respectively for the set of expected value and variance optimal solutions for problem.

A shortcoming of the above defined solution concepts is that, the expected value and the variance do not exhaust the information contained in the distributions of involved random variables. To overcome this drawback, other solution concepts have been proposed. We discuss some of them in the next three subsections [13].

Tammer and Minimum Risk Optimalities and Optimality in Probability:

y_* is a Tammer β -optimal solution for Problem,

If there is no y ∈ B such that

Q (α: b (α) y ≤ b (α)y*) ≥1-β

And Q (α: b (α) y < b (α) y_*) > 0

When B is a transformation of B *(α)* obtained through technique of stochastic optimization. Here β is a probability level pre-defined by the Decision maker.

For details on this solution concept, we invite the reader to consult [36].

Example: *y* $_*$ is an β-minimum risk optimal solution

If *y* $_*$ is an optimal solution max Q (b (α) *y*≤β)

When D is a transformation of B (α) obtained through technique of stochastic optimization.

Where β is an aspiration level a-priori fixed by the Decision maker.

Example: *y* $_*$ is a p-optimal solution in probability.

If there is $\beta_* \in \mathbb{R}$ such that (y_*, β_*) is optimal for the program:

min β
(*y*, β) ∈ B×S
Subject to
Q (b (α) *y*≤β) = P

When B is a transformation of B (α) obtained through technique of stochastic optimization.

Where P is a probability level Pre-defined by the decision maker.

Expected value and variance Efficiencies:

Consider the following deterministic different type of objective Programs:

min
y ∈ B {F (b' (α)) *y*……..F (b^m (α)) *y*}
min
y ∈ B {Z (b' (α)) y…….Z (b^m (α)) *y*}
min
y ∈ B{F(b'(α))y,……F(b^m(α))y, σ(b'(α))y……..σ(b^m(α))y}

Where σ stand for the standard deviation.

Example: y_* is called an expected value, a variance or an expected value/standard deviation efficient salutation.

If y $_*$ is efficient respectively, when B is a transformation of B (α) obtained though technique of stochastic optimization model.

The concepts of expected value weak efficiency, variance weak efficient and those of expected value proper efficiency, variance proper efficient and expected value/standard deviation proper efficiency are obtained by replacing "efficiency by weak efficient" and by "proper efficiency respectively"

In the sequel $\phi^U{}_F$ ($\phi^Q{}_F$), $\phi^Q{}_F$ ($\phi^Q{}_Z$) and $\phi^U{}_{F/\sigma}$ ($\phi^Q{}_{F/\sigma}$)

Denoted the set of expected value weakly efficient salutation, variance weakly efficiency salutation and expected value/standard deviation weakly efficient salutation for program respectively.

Minimum Risk Efficiency and Efficiency in Probabilities:

Minimum risk efficiency is defined as follows:

Example: y_* is an $(\beta_1.....\beta_m)$ minimum risk efficient salutation,

If y_* is efficient for the different type of objective program;

$\max \{Q (b' (\alpha) y \leq \beta_1\}$

$y \in B$

When B is a transformation of B (α) obtained though technique of stochastic optimization.

Here $\{(\beta_1........\beta_m)\}$ are aspiration level a-priori fixed by the decision maker.

As in the case of expected value efficiency, the concept of $(\beta_1......\beta_m)$ minimum risk weak efficiency and $(\beta_1......\beta_m)$ minimum risk proper efficient may be obtained by respectively replacing "efficiency by weak efficiency or proper efficiency in the above definition.

In what follows ψ_{HS} (β1……βm), ψ^{v}_{HS} $(\beta_1......\beta_m)$ and ψ^{Q}_{HS} $(\beta_1......\beta_m)$ denoted the sets of $(\beta_1........\beta_m)$ minimum risk efficient salutation, (β1…….βm) minimum risk weakly efficient solution and (β1……βm) minimum risk property efficient salutation.

Example: y_* is a $(P_1.....P_m)$ efficient salutation in probability, if there is

$B_* = (\beta_{1*}......\beta_{*}m)$ such that (y_*, β_*) is efficient for the mathematical program:

$\min (\beta_1........\beta_m)$

$(y, \beta) \in B \times S^m$

Subject to

$Q (b_m (\alpha) y \leq \beta_m) \geq P_m$, m= 1…….M

When B is a transformation B (α) obtained through technique of stochastic optimization model. Where $P_1......P_m$ are probability levels that are a-priori fixed by the Decision maker.

An interested reader may consult for a thorough discussion on this efficient concept [37].

Concepts of $(P_1.......P_m)$ weak efficiency in probability and $(P_1........P_m)$ proper efficiency in probability may also be obtained in a way similar to the one in which minimum risk weak and proper efficiencies were obtained.

From now on ψ_{KT} $(P_1.........P_m)$, ψ^{U}_{KT} $(P.......P_m)$ and ψ^{Q}_{KT} $(P_1......P_m)$ denote the set of $(P_1........P_K)$ efficient salutation in probability, and $(P_1....P_m)$ properly efficient salutation in probability for program respectively.

Most stochastic constraint transformation yield non convexity on resulting deterministic feasible sets. This precludes the application of existing powerful convex optimization algorithms. It is therefore, relevant to know when a deterministic counterpart of B (α) is convex.

Case 1.

B" (0), B" (1), Bi (0); i = 1……….P, Bi (1);

i =1……..P and D" are convex sets.

Case 2.

c (α) is a fixed matrix with maximal rank. Then

B_i (B_i) = {Y ∈ ℝ ^{q}: G; (c_iy) ≥β_i i=1……..P}

are convex for every probability distribution G_i of d_i (α)

Case 3.

Assume that the probability space under consideration is discrete, that is Ω= {α_1………α_J} and Q (α_l) = q_1>0, j= 1……….J.

Let β^*_l= max (1-q: 1≤j≤J) then the set β_l (β_l) is convex for any β_l>β_l^* and B" (β) is convex for any β>β^* where β^*_l are real numbers.

Case 4.

Suppose that the probability space under consideration is Ω= {α_1……..αJ} and suppose that q_l = Q (α_l)>0

If and only if l ∈M= {1…….Q}

Assume also that only one element j_o ∈M exists

Such that q_{jo}= $\min_{ql}$

j ∈ M

then the set B" (β) and B_l" (β) are convex for every β>1-q_{j1}

Where q_{j1}=min ql

j ∈ (M/{j_o})

Case 4.

1) $\phi_F \cap \phi_z \subset \phi_{F/\sigma}$

2) $\phi_F \cup \phi_z \subset \phi^U{}_{F/\sigma}$

3) $\phi_F{}^U \cup \phi_Z{}^U \subset \phi^U{}_{F/\sigma}$

Case 5.

Assume that probability distributions of the random vector b' (α)…..b_m (σ) are continuous and strictly increasing.

Then for any

(β_1…….β_m) ∈ $\mathbb{R}^m$, y_* ∈ ψ_{HS} (β_1……….β_m)

And only if y_* ∈ ψ_{KT} (P_1……...P_m)

Where P_m=Q (b_m (α) y≤β_m); m∈ {1…….M}

Case 6.

If y_* is an expected value optimal salutation, then y_* is an expected value properly efficient salutation

That is

$$\psi_F \subseteq \phi_F{}^Q$$

Example: If D is a convex set and F (y^m (α)) y; m=1………m

are convex functions, then y_* is an expected value properly efficient salutation for the different type of objective program,

If and only if,

y_* is an expected value optimal salutation that is

$$\phi^Q{}_F = \psi_F$$

Methodological Approaches for solving different type of objective stochastic Linear Programs:

This ideas discussed in the previous sections have served as guidelines in implementing efficient techniques for solving different type of Stochastic Linear Programming Problems.

Examples: using stochastic approach for this method assumptions should be

c (α), *i* =1………p; d (α) and b^m

(α), m=1……….M

are normally distributed random vector λ_m

m=1………M are strictly positive real numbers in the interval

(0, 1] such that $\Sigma^M_{m=1} \lambda_m=1$

Moreover, the following notations are used:

1) l_i (α, y) = c*i* (α)y-di (α), i=……….p

2) ϕ denote the cumulative distribution function of the standard normal random Variable

3) r_1 and r_2 are weights associated with the expected value and the standard deviation of b (α) respectively.

4) β= (β_1………β_p) where β_i *i*=1…….p are probability levels prescribed by the decision maker for constraints satisfaction.

This method is as following:

λ_m, m=1……….M; y^m (α), m=1…………M;

L_i (α, y), *i*=1……..p; β_i, i=1………p

Find b (α) = $\Sigma^M_{m=1} \lambda_m b^m$ (α)

Replace B (α) by

B^z= {y∈ s^q: F (l_i (α, y)) +$\phi^{-1}(\beta_i)$ σ (l_i (α, y)) ≤0, *i*=1………..p; y≥0}

Solve the mathematical program:

min(t_1 F (b (α)) +t_2 σ (b(α))

y∈B^z

Let y_* be a solution

This algorithm transform the original problem into a single objective problem that has been put in the deterministic, using the expected value model approach.

The salutation y_* obtained is an expected value/standard deviation efficient salutation.

Other techniques closely related to the stochastic approach for solving problem include, decomposition method, chance-constrained method, simulation based techniques two stage method and multistage method [34].

Multi objective Approach:

Here we outline a method within the multiobjective approach method,

We need λ_m, m=1……..M

Such that λ_m>0, $\Sigma^M_{m=1}$, λ_m=1

Example: Read λ_m, m=1………M: b^m (α) m=1………M, c (α): d (α)

Replace B (α) by

B'= {y∈ Sq: F(c (α)) y-F(d (α)) y-Fd(α)) ≤0:y≥0}

Find

F (b' (α))……..F (b^M (α))

Solve the mathematical program:

min ($\Sigma^M_{m=1} \lambda_m$ F(b^m (α)y)

y_* be a salutation obtained is an expected value efficient salutation for Type of Optimization Model as defined.

Hybrid Approach:

This method is based on the assumption.

The following notation are used in the sequel.

1) δ^+_w, w=1………W; v=1………V, δ^+_y, δ^-_y y=1…….y denote positive, negative and two sided deviations from targets

i_w, w=1……….w; i_v v=1……..V; i_y; y=1………y respectively.

w, v and y are respectively the total number of positive, native and two-sided deviations from targets i_w, i_v and i_y.

2) β_w, w=1………..W; β_v, v=1……….v; β_y y=1………y are *Probability level a-priori fixed by the decision maker. Discussion of other method for solving Type of Optimization method based on the different type of approach [38-44].*

Method of hybrid Approach:

w, v, y, i_w, β_w, w=1…….w; i_v, β_v, v=1………v;

i_y, β_y, y=1……..y; b^m (α)

m=1……….M; l_i (α, y), *i*=1………b

Put B (α) in following form:

B^{vi}={y∈ ℝq: $F(b^w(\alpha))y+ \phi^{-1}(\beta_w)\sigma(b_w(\alpha))y-i_w-\delta^+_w\le 0$,

W=1…….W; $F(b^v(\alpha))y+\phi^{-1}(1-\beta_v)\sigma(b^v(\alpha))y-i_v+g^+_v\le 0$,

v=1……..v; $F(b^y(\alpha))y+\phi^{-1}(\frac{1-\beta y}{2})\sigma(b^m(\alpha))y-i_y\le 0$

y=1…….y; $F(l_v(\alpha,y))+\phi^{-1}(\beta i)\sigma(l_i(\alpha,y))\le 0$

i=1………..p, {$\delta^+_w\ge 0$, $\delta^+_v\ge 0$, x≥0}

It is clear that method combines the goal programming technique for solving different type of objective program with the chance-constrained method for solving a stochastic optimization problem [45-47].

Applications:

I. Applications of the stochastic Approach

II. Application along the different type of Approach

III. Application within the hybrid Approach

Discussion and Conclusions:

In this paper we have presented the main principle of Different type of Optimization Model. We have also indicated that there are concrete realizations in this field. We have also discussed approaches and limitations of Types of Optimization Model. To cater the best for a broad readership, the paper has the following distinctive features: The literature is rich in models using the different type of approach. Decision maker should be able to consider different objective functions and incorporate imprecision into the model. Owing to the complexity of such problems, it is the best to couple different techniques in an appropriate way to solve them.

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