YOUNG RESEARCHER
ISSN: 2277-7911 Impact Factor: 5.077 (SJIF 2021)
Peer-Reviewed, Refereed, Indexed Journal
Volume - 12, Issue - II, Apr-May-June 2023

A SURVEY OF TOPOLOGICAL VECTOR SPACES FOR ANALYTIC FUNCTIONS

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DOI - 10.5281/zenodo.10426589

ABSTRACT:

*In a work that we just published, we created a new space of analytic functions in the unit disc that we termed W(1). This space is the union of a few other spaces called X'(a), and it spans the range 0 to 1. We also demonstrated that if f(z) E *(I), then f(z) can be recovered despite the limitation of its distributional boundary values to an open arc of the unit circle. This was another one of our accomplishments. In this article, we are going to explore some of the attributes of the spaces that we utilised in our prior work. Specifically, we are going to focus on the percentage a. On the other hand, this will be done in a manner that is more broad. In point of fact, for any positive and smooth. Concave sub additive function w(x) on [0, a,) with d x) = O(x) as x + x, we will create a Fréchet space W(o(x)) of analytic functions on the unit disc. This space will be in the form of a disc. The many characterizations, multipliers, and dual spaces of X(w) will be investigated as part of this project. We acquire the space X' as a special case if W(X=) x u is true for any value of a between 0 and I. (a).*

Keywords: Topological vector spaces of holomorphic functions, Taylor coefficients, Multipliers, linear functional.

INTRODUCTION:

The primary objective of this discussion is to define complicated analytic functions and to describe some of their most essential characteristics. If a function f(z) has a complex derivative f(0), then the function is said to be analytic (z). Calculus with one variable should have familiarised you with the fundamental principles behind the computation of derivatives. However, in comparison to real differentiable functions, complex analytic functions allow for a far wider range of inferences to be derived regarding their behaviour.

THE DERIVATIVE: PRELIMINARIES:

In calculus we defined the derivative as a limit. In complex analysis we will do the same.

$$f'(z) = \lim_{\Delta z \to 0} \frac{\Delta f}{\Delta z} = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}.$$

Before giving the derivative our full attention we are going to have to spend some time exploring and understanding limits. To motivate this we'll first look at two simple examples – one positive and one negative.

Example 2.1. Find the derivative of $f(z) = z^2$.

Solution: We compute using the definition of the derivative as a limit.

$$\lim_{\Delta z \to 0} \frac{(z+\Delta z)^2 - z^2}{\Delta z} = \lim_{\Delta z \to 0} \frac{z^2 + 2z\Delta z + (\Delta z)^2 - z^2}{\Delta z} = \lim_{\Delta z \to 0} 2z + \Delta z = 2z.$$

That was a positive example. Here's a negative one which shows that we need a careful understanding of limits.

Example 2.2. Let $f(z) = \overline{z}$. Show that the limit for $f'(0)$ does not converge.

Solution: Let's try to compute $f'(0)$ using a limit:

$$f'(0) = \lim_{\Delta z \to 0} \frac{f(\Delta z) - f(0)}{\Delta z} = \lim_{\Delta z \to 0} \frac{\overline{\Delta z}}{\Delta z} = \frac{\Delta x - i\Delta y}{\Delta x + i\Delta y}.$$

Here we used $\Delta z = \Delta x + i\Delta y$.

Now, $\Delta z \to 0$ means both Δx and Δy have to go to 0. There are lots of ways to do this. For example, if we let Δz go to 0 along the x-axis then, $\Delta y = 0$ while Δx goes to 0. In this case, we would have

$$f'(0) = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x} = 1.$$

On the other hand, if we let Δz go to 0 along the positive y-axis then

$$f'(0) = \lim_{\Delta y \to 0} \frac{-i\Delta y}{i\Delta y} = -1.$$

The limits don't agree! The problem is that the limit depends on how Δz approaches 0. If we came from other directions we'd get other values. There's nothing to do, but agree that the limit does not exist.

PROPERTIES OF CONTINUOUS FUNCTIONS:

Since continuity is defined in terms of limits, we have the following properties of continuous functions.

Suppose $f(z)$ and $g(z)$ are continuous on a region A. Then

- $f(z) + g(z)$ is continuous on A.
- $f(z)g(z)$ is continuous on A.
- $f(z)/g(z)$ is continuous on A except (possibly) at points where $g(z) = 0$.
- If h is continuous on $f(A)$ then $h(f(z))$ is continuous on A.

Using these properties we can claim continuity for each of the following functions:

- e^{z^2}
- $\cos(z) = (e^{iz} + e^{-iz})/2$
- If $P(z)$ and $Q(z)$ are polynomials then $P(z)/Q(z)$ is continuous except at roots of $Q(z)$.

APPENDIX: LIMITS:

The fundamental principle that underpins limitations is one that is not too complicated. Nevertheless, throughout the 19th century, mathematicians were concerned about the lack of rigour in their field. As a result, they set out to provide a solid foundation for limits and analysis by carefully defining terms and providing proofs. In this supplemental material, we will provide you with the official definition and explain how it relates to the overall concept. After version 18.04, we won't need quite this degree of formality. Nevertheless, it's comforting to be aware that the foundations are rock-solid, and it's possible that some pupils may find this information intriguing.

The boundaries of sequences Intuitively, we say that a series of complex numbers, z1, z2,... converges to a, if for large n, zn is extremely near to a. This is because convergent sequences tend to be more stable than divergent ones. To be a little bit more specific, if we place a little circle of radius around a, then ultimately the sequence should remain inside the circle. This is what we mean by "staying within the circle." Let's say we're going to refer to this as the sequence that the circle is capturing. This must be true for any circle, regardless of how tiny it is, but it may take far longer for a smaller circle to 'catch' the sequence than it would for a larger circle. This is shown in the figure that can be seen below. The sequence is laid down in the form of a string that follows the curve that is depicted. The sequence is captured by the larger circle with a radius of 2 at the time n = 47, however the sequence is not captured by the smaller circle until the time n = 59. It is important to note that z25 is located inside the wider circle; nevertheless, due to the fact that following points are located outside the circle, we cannot declare that the sequence is recorded at n = 25.

Definition. The sequence $z_1, z_2, z_3, \ldots$ converges to the value a if for every $\epsilon > 0$ there is a number N_ϵ such that $|z_n - a| < \epsilon$ for all $n > N_\epsilon$. We write this as

$$\lim_{n\to\infty} z_n = a.$$

Again, the definition just says that eventually the sequence is within ϵ of a, no matter how small you choose ϵ.

Example 2.20. Show that the sequence $z_n = (1/n + i)^2$ has limit -1.

Solution: This is clear because $1/n \to 0$. For practice, let's phrase it in terms of epsilons: given $\epsilon > 0$ we have to choose N_ϵ such that

$$|z_n - (-1)| < \epsilon \text{ for all } n > N_\epsilon$$

One strategy is to look at $|z_n + 1|$ and see what N_ϵ should be. We have

$$|z_n - (-1)| = \left|\left(\frac{1}{n} + i\right)^2 + 1\right| = \left|\frac{1}{n^2} + \frac{2i}{n}\right| < \frac{1}{n^2} + \frac{2}{n}$$

So all we have to do is pick N_ϵ large enough that

$$\frac{1}{N_\epsilon^2} + \frac{2}{N_\epsilon} < \epsilon$$

Since this can clearly be done we have proved that $z_n \to i$.

This was clearly more work than we want to do for every limit. Fortunately, most of the time we can apply general rules to determine a limit without resorting to epsilons!

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